The Jury Test

for stability of a discrete-time system

Given a transfer function $H(z) = \frac{b(z)}{a(z)}$, the system is stable if and only if all roots of $a(z) = a_0 z^n + a_1 z^{n-1} + \ldots + a_n$ are inside the unit circle.

To use the Jury test, begin by multiplying $a(z)$ by -1 if necessary to make a_0 positive. Then form the following array:

$$
\begin{array}{ccccc}
a_0 & a_1 & \ldots & a_{n-1} & a_n \\
a_n & a_{n-1} & \ldots & a_1 & a_0 \\
b_0 & b_1 & \ldots & b_{n-1} & \cdot \\
b_{n-1} & b_{n-2} & \ldots & b_0 & \cdot \\
c_0 & c_1 & \ldots & \cdot & \cdot \\
c_{n-2} & c_{n-3} & \ldots & \cdot & \cdot
\end{array}
$$

Third-row entries are based on second-order determinants divided by a_0 of the first two rows, starting with the first and last columns, then the first and second-to-last columns, etc.:

$$b_0 = a_0 - \tfrac{a_n}{a_0} a_n$$

$$b_1 = a_1 - \tfrac{a_n}{a_0} a_{n-1}$$

$$b_k = a_k - \tfrac{a_n}{a_0} a_{n-k}$$

The fourth row is made by reversing the third row, and the fifth row is given by

$$c_k = b_k - \tfrac{b_{n-1}}{b_0} b_{n-1-k}$$

If all the terms in the first columns of the odd rows are positive, the polynomial $a(z)$ is stable.